BVKT1

3. Schulaufgabe im Fach Mathematik

am 10.7.13

1.1)

$A(t) = A_0 \cdot b^t$

$\Leftrightarrow \quad 9{,}5 = 1{,}5 \cdot b^{10} \quad | : 1{,}5$

$\Leftrightarrow \quad \frac{19}{3} = b^{10} \quad | \sqrt[10]{\ }$

$\Leftrightarrow \quad b \approx 1{,}203$

$\underline{\underline{A(t) = 1{,}5 \cdot 1{,}203^t}}$

1.2)

$A(t) = 7$

$\Leftrightarrow \quad 7 = 1{,}5 \cdot 1{,}203^t \quad | : 1{,}5$

$\Leftrightarrow \quad \frac{14}{3} = 1{,}203^t \quad | \ln$

$\Leftrightarrow \quad \ln\left(\frac{14}{3}\right) = t \cdot \ln(1{,}203) \quad | : \ln(1{,}203)$

$\Leftrightarrow \quad \underline{\underline{t}} = \frac{\ln\left(\frac{14}{3}\right)}{\ln(1{,}203)} \approx 8{,}3 \rightarrow \underline{\underline{9 \text{ Tagen}}}$

1.3)

$\cancel{A_0} \cdot 1{,}203^t = \cancel{A_0} \cdot e^{k \cdot t} \quad | \ln$

$\cancel{t} \cdot \ln(1{,}203) = k \cdot \cancel{t}$

$\underline{\underline{k \approx 0{,}185}}$

1.4)

$m_{Sekante} = \frac{\Delta y}{\Delta x} = \frac{8}{10} = 0{,}8$

1.6) $M(t) = 9{,}5 \cdot 0{,}85^{t-10}$ für $t > 10$

~~0,85~~ 1.5) $t^* \approx 5{,}75$

Aufgabe 2)

$$\sin\left(\frac{\pi}{3} \cdot x - 4\right) = 0{,}5$$

$$u_{TR} = \sin^{-1}(0{,}5) = \frac{1}{6}\pi$$

$$\frac{\pi}{3} \cdot x_{1k} - 4 = \frac{1}{6} \cdot \pi + k \cdot 2\pi \quad | +4$$

$$\frac{\pi}{3} \cdot x_{1k} = \frac{\pi}{6} + 4 + k \cdot 2\pi \quad | \cdot \frac{3}{\pi}$$

$$x_{1k} = \frac{\cancel{\pi} \cdot 3}{\cancel{\pi} \cdot 6} + \frac{12}{\pi} + \frac{k \cdot 2\cancel{\pi} \cdot 3}{\cancel{\pi}}$$

$$x_{1k} = \frac{1}{2} + \frac{12}{\pi} + 6k$$

$$v_2 = \pi - u_{TR}$$

$$v_2 = \pi - \frac{1}{6}\pi = \frac{5}{6}\pi$$

$$\frac{\pi}{3} \cdot x_{2k} - 4 = \frac{5}{6}\pi + k \cdot 2\pi \quad | +4$$

$$\frac{\pi}{3} \cdot x_{2k} = \frac{5}{6}\pi + 4 + k \cdot 2\pi \quad | \cdot \frac{3}{\pi}$$

$$x_{2k} = \frac{5\cancel{\pi} \cdot 3}{6\cancel{\pi}} + \frac{12}{\pi} + \frac{k \cdot 2\cancel{\pi} \cdot 3}{\cancel{\pi}}$$

$$x_{2k} = 2{,}5 + \frac{12}{\pi} + 6k$$

Aufgabe 3)

$$f(x) = a \cdot \cos(bx + c)$$
$$\Rightarrow a \cdot \cos(b(x + d)$$

$a = 2$;

$b = \frac{2\pi}{p} = \frac{2\pi}{48} = \frac{1}{2}\pi$; $d = +\frac{1}{2}$

$$f(x) = 2 \cdot \cos\left(\tfrac{1}{2}\pi\left(x - \tfrac{1}{2}\right)\right)$$
$$\underline{\underline{f(x) = 2 \cdot \cos\left(\tfrac{1}{2}\pi \cdot x - \tfrac{1}{4}\pi\right)}}$$

Aufgabe 4)

$$p_a(x) = a \cdot (x+1)(x-5)$$
$$f(x) = \ln(p_a(x))$$

4.1) $P(0|1{,}25)$ in p_a

$\cancel{p_a(0) = a(}$ $\quad 1{,}25 = a \cdot (0+1)(\cancel{x}0-5)$

$\Leftrightarrow 1{,}25 = -5a$

$\Leftrightarrow \underline{\underline{a = -0{,}25}}$

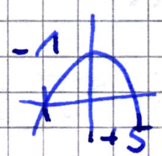
4.2) $f(x) = \ln(-0{,}25 \cdot (x+1)(x-5))$

$\ln(-0{,}25(x^2 - 4x - 5)$

$\ln(-0{,}25x^2 + 1x + \frac{5}{4})$

D_{max}: $\quad p_{-0{,}25} > 0$

$\underline{\underline{D_{max} = [-1;\ 5]}}$



$x \to -1$: $\quad p(x) \to 0$; $\underline{\underline{f(x) \to -\infty}}$

$x \to 5$: $\quad p(x) \to 0$; $\underline{\underline{f(x) \to -\infty}}$

senkrechte Asymptoten: $\underline{\underline{x = -1}}$; $\underline{\underline{x = 5}}$

4.3) $\cancel{p}\ \cancel{f_{0,25}}\ \cancel{\ln}\ f(x) = \ln\left(-0{,}25x^2 + 1x + \frac{5}{4}\right)$

NST: $\quad p\cancel{f}(x) = 1$

$\cancel{\ln}\ -0{,}25x^2 + 1x + \frac{5}{4} = 1 \quad | -1$

$-0{,}25x^2 + 1x + \frac{1}{4} = 0$

$$x_{1/2} = \frac{-1 \pm \sqrt{1 - 4\cdot(-0{,}25)\cdot\frac{1}{4}}}{2\cdot(-0{,}25)}$$

$$x_{1/2} = \frac{-1 \pm \frac{\sqrt{5}}{2}}{-\frac{1}{2}} \quad \nearrow x_1 = 2-\sqrt{5} \approx -0{,}24 \quad \searrow x_2 = 2+\sqrt{5} \approx 4{,}24$$

$N_1(2-\sqrt{5}/0);\ N_2(2+\sqrt{5}/0)$

$f(0) = \ln\left(-0{,}25x^2 + 1x + \frac{5}{4}\right)$

$f(0) = \ln\left(\frac{5}{4}\right) \approx 0{,}22$

$S_y\left(0 \mid \ln\left(\frac{5}{4}\right)\right)$

$y_S = -0{,}25\cdot 2^2 + 2 + \frac{5}{4} = \frac{9}{4}$

$\cancel{\frac{9}{4} = \ln}\quad f(2) = \ln\left(\cancel{2}\ \frac{9}{4}\right) \approx 0{,}81$

$HOP\left(2 \mid \ln\left(\frac{9}{4}\right)\right)$

4.4 $\quad G(p_a)$ muss unterhalb der Geraden $y=1$ verlaufen

$y_S < 1 \Rightarrow p_a(x_S) < 1 \Rightarrow a(2+1)(2-5) < 1 \Leftrightarrow -9a < 1$

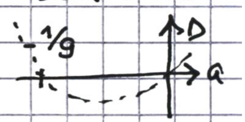
$\Leftrightarrow a > -\frac{1}{9}$; außerdem $a<0 \Rightarrow a \in \left]-\frac{1}{9}; 0\right[$

Alternativ: $p_a(x) = 1$ darf keine Lsg haben:

$a(x^2-4x-5) = 1 \Leftrightarrow ax^2 - 4ax - 5a - 1 = 0$

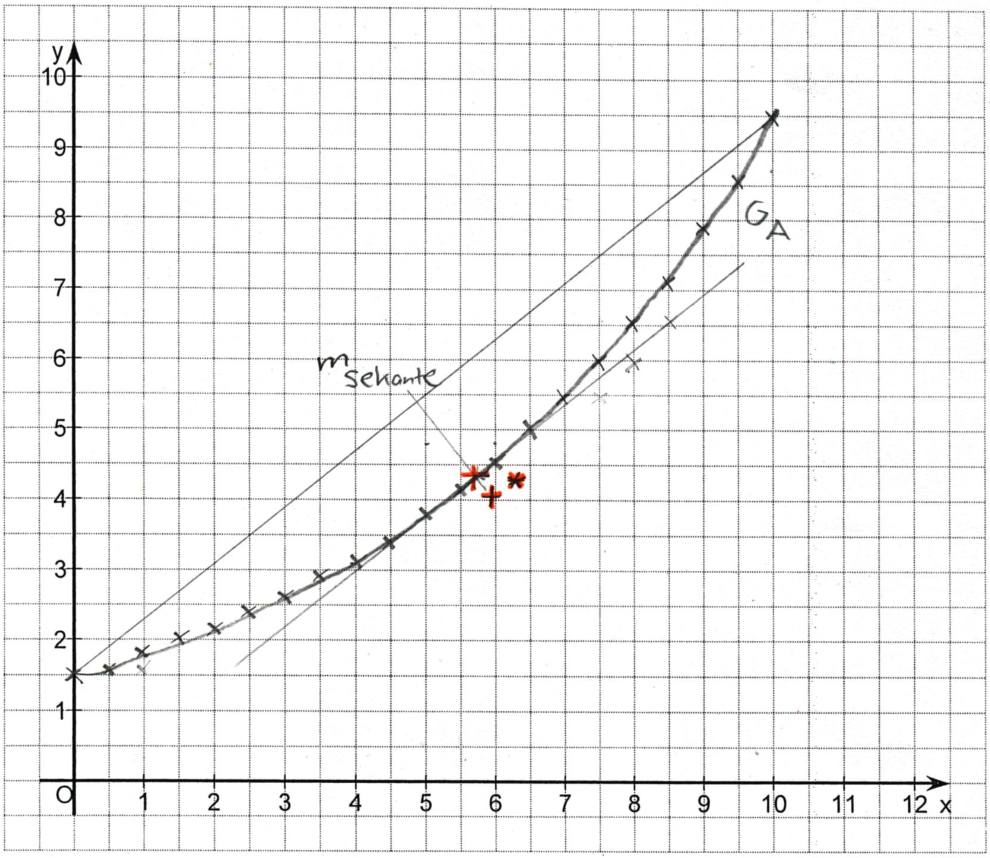
$D = (-4a)^2 - 4\cdot a(-5a-1) = 16a^2 + 20a^2 + 4a = 36a^2 + 4a$

$D = 4a(9a+1) = 0 \Rightarrow a_1 = 0;\ a_2 = -\frac{1}{9}$

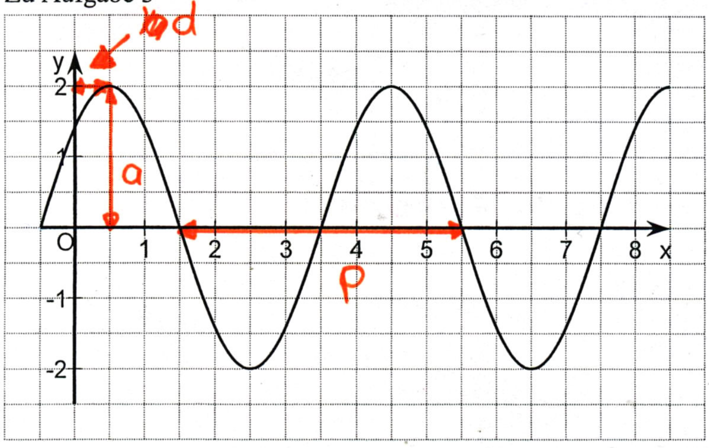


$D < 0$ für $a \in \left]-\frac{1}{9}; 0\right[$

Zu Aufgabe 1



Zu Aufgabe 3



Zu Aufgabe 4